DIFFUSION OF SUPERINTENSE PULSED MAGNETIC FIELDS

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The phenomena accompanying the diffusion of superintense pulsed magnetic fields in metals are studied. Solutions are obtained for the diffusion of a constant superintense magnetic field into a half-space with a plane boundary and for the diffusion of an axial concentrated magnetic field. The limiting magnetic fields which can be obtained in various experimental devices are found. The results are compared with experiment.

Existing theories for the superheating of the skin layer of a metal are based on the assumption that the conductivity is either independent of the temperature T [1] or has the dependence [2]

$$\sigma = \sigma_0 / (1 + \beta T),$$

where β is the temperature coefficient of the resistance, and σ_0 is the conductivity at T = 0. However, Joule heating during the diffusion of a superintense magnetic field evaporates the metal. Since pure metals are elemental (monatomic) substances, they do not dissociate in the evaporated state, and can have a nonvanishing conductivity only as a result of ionization. During pulsed evaporation of a metal heated to its boiling temperature T_*, however, ionization of its vapor is prevented by the high density. An example of the evidence for the vanishing conductivity of a highly dense evaporated metal is the "current pause" [3] during the electrical explosion of wires.

A study is therefore in order of the effect on magnetic field diffusion of the vanishing of the conductivity when a metal is heated to its boiling temperature and evaporated.

1. Diffusion of a constant superintense magnetic field into a half-space. We consider the diffusion of a magnetic field H(x, t) into the half-space x > 0 where H(x, 0) = 0 and H(0, t) = H_0, where H_0 is the constant superintense magnetic field.

We assume the conductivity of the metal in the half-space satisfies

$$\sigma = \text{const} \ \ (Q < Q_*), \qquad \sigma = 0 \ \ (Q = Q_*), \tag{1.1}$$

where Q(x, t) is the heat evolved per unit volume at cross section x before time, t, Q(x, 0) = 0, and Q_* is the heat required to heat unit volume of the metal from its initial temperature to its boiling temperature and to completely evaporate it. We thus assume that the absorption of Q_* is accompanied by a change in physical state: the metal converts from a conductor to a dielectric.

If the magnetic field is large enough, the surface at which the conductivity vanishes (the phase-transition surface) penetrates into the half-space according to x = ξ(t). At the phase-transition surface, the following condition is satisfied:

$$Q(\xi, t) = Q_* \tag{1.2}$$

The thermal conductivity of molten metals at all temperatures below the critical temperature [4] follows the Wiedemann-Franz-Lorentz law

$$\varkappa = L\sigma T \tag{1.3}$$

where L = $2.44 \cdot 10^{-8}$ W · ohm/deg^2 is the Lorentz constant.

In that part of the half-space in which the transition has not yet occurred, heat conduction is described by

$$\frac{\partial Q}{\partial t}=\frac{\partial}{\partial x}\left(\varkappa\frac{\partial T}{\partial x}\right)+\frac{1}{\sigma}\left(\frac{\partial H}{\partial x}\right)^2, \tag{1.4}$$

which takes into account Joule heating. For the region below the boiling point T_*, the order of magnitude of the quantities on the right-hand side of this last equation can be evaluated from (1.3):

$$\frac{\partial}{\partial x}\left(\varkappa\frac{\partial T}{\partial x}\right)\Big/\frac{1}{\sigma}\left(\frac{\partial H}{\partial x}\right)^2\sim\frac{L\sigma T^2}{H^2}\ll 1$$

for $\sigma < 5\cdot 10^7$ ohm$^{-1}\cdot$m^{-1}, $T \ll T_* = 2600°C$, and $H > 10^8$ a/m (1 MOe).

Heat conduction can therefore be neglected in this region during the diffusion of an intense magnetic field. In the region in which the evaporation is occurring, the temperature remains constant until the metal is completely evaporated; i.e., the first term on the right of (1.4) vanishes, and there is no heat conduction. Heat conduction can therefore be neglected throughout the region in which the phase transition has not occurred; Eq. (1.4) becomes

$$\frac{\partial Q}{\partial t}=\frac{1}{\sigma}\left(\frac{\partial H}{\partial x}\right)^2 \qquad (\xi<x<\infty). \tag{1.5}$$

During the diffusion of the magnetic field, the displacement currents outside the conductors can also be neglected, so

$$H(x,\ t)=H_0\ \ (0\leqslant x\leqslant\xi). \tag{1.6}$$

According to (1.1), the conductivity is constant in that part of the half-space in which the transition has not occurred, and the diffusion equation for the magnetic field is

$$\frac{\partial H}{\partial t}=\eta^2\frac{\partial^2 H}{\partial x^2} \qquad \left(\xi<x<\infty,\ \eta^2=\frac{1}{\mu\sigma}\right), \tag{1.7}$$

where η^2 is the diffusion coefficient for the magnetic field, and μ is the magnetic permeability.

The problem of the diffusion of a constant superintense magnetic field H_0 into a half-space with a conductivity satisfying (1.1) thus reduces to system (1.5), (1.7).

We seek a solution of Eq. (1.5) in the form

$$H(x,t)=A+B\Phi\left(\frac{x}{2\eta\sqrt{t}}\right) \qquad \left(\Phi(z)=\frac{2}{\pi}\int_0^z e^{-\tau^2}d\tau,\ A,\ B=\mathrm{const}\right). \tag{1.8}$$

From the initial conditions for the magnetic field intensity and condition (1.6) at the phase-transition boundary, we find

$$H_0=A\,[1-\Phi(\xi/2\eta\sqrt{t})]. \tag{1.9}$$

Since this latter equation holds for all t, we have

$$\xi/\sqrt{t}=\alpha=\mathrm{const}. \tag{1.10}$$

Equation (1.10) describes the motion of the phase-transition boundary. From (1.9) and (1.10), we find

$$A=H_0/[1-\Phi(\alpha/2\eta)]. \tag{1.11}$$

Using solution (1.8) and (1.5), we find, after some simple calculations,

$$Q(x,\ t)=\frac{\mu}{\pi}A^2E_1\left(\frac{x^2}{4\eta^2 t}\right)+\varphi(x) \qquad \left(E_1(z)=\int_z^\infty e^{-\tau}\tau^{-1}d\tau\right), \tag{1.12}$$

where $\varphi(x)$ is an arbitrary function. From (1.11), (1.12), the initial condition $\varphi(x) = 0$, and the condition at the phase-transition boundary, we find

$$Q_* = \frac{\mu}{\pi} H_0^2 \frac{E_1(\alpha^2/2\eta^2)}{[1-\Phi(\alpha/2\eta)]^2}. \tag{1.13}$$

Using the asymptotic expressions for the functions $\Phi(z)$ [5] and $E_1(z)$ [6], we find the limiting magnetic field H_{max} as $z \to \infty$:

$$H_{max} = (2Q_*/\mu)^{1/2} \quad (\alpha/2\eta \to \infty). \tag{1.14}$$

If the field H_0 reaches H_{max}, its diffusion rate is infinite. This means that fields greater than H_{max} cannot be maintained in any type of experimental apparatus with plane boundaries.

2. **Diffusion of a constant, superintense, axial magnetic field.** We assume that at time $t = 0$ the superintense magnetic field H_0 is concentrated at the line $r = 0$, where r is a cylindrical coordinate. We consider the diffusion of the field into a space with a conductivity satisfying condition (1.1). We also assume that some external apparatus holds the magnetic field intensity constant at H_0 at the line $r = 0$ during the diffusion.

Since the same physical processes occur in this case as during diffusion into the half-space, the problem reduces to the solution of the following system of equations in cylindrical coordinates:

$$\frac{\partial H}{\partial t} = \frac{\eta^2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right), \quad \frac{\partial Q}{\partial t} = \frac{1}{\sigma}\left(\frac{\partial H}{\partial r}\right)^2 \qquad (\xi < r < \infty), \tag{2.1}$$

where $r = \xi(t)$ is the law of motion for the phase-transition boundary. The initial conditions are

$$Q(r, 0) = 0, \quad H(r, 0) = 0; \tag{2.2}$$

the boundary conditions are

$$H(r, t) = H_0 \quad (0 \leqslant r \leqslant \xi), \quad Q(\xi, t) = Q_*. \tag{2.3}$$

We seek a solution of the first equation in (2.1) in the form

$$H(r, t) = A + BE_1(r^2/4\eta^2 t) \quad (AB = \text{const}). \tag{2.4}$$

It follows from the second initial condition in (2.2) that $A = 0$, and it follows from boundary condition (2.3) that

$$H_0 = BE_1(\xi^2/4\eta^2 t); \tag{2.5}$$

then the law of motion of the phase-transition boundary is of the same form as for the magnetic field diffusion into the half-space:

$$\xi / \sqrt{t} = \alpha = \text{const}. \tag{2.6}$$

From (2.5) and (2.6) we find

$$B = H_0 / E_1(\alpha^2/4\eta^2). \tag{2.7}$$

The solution of the second equation in (2.1) becomes

$$Q(r, t) = \frac{2\mu}{\pi} B^2 \frac{E_2(r^2/2\eta^2 t)}{r^2/2\eta^2 t} + \psi(r) \quad \left(E_2(z) = \int_1^\infty e^{-zu} u^{-2}\, du\right), \tag{2.8}$$

where $\psi(r)$ is an arbitrary function. We see from initial condition (2.2) for Q that $\psi(r) = 0$; from (2.7), (2.8), and boundary condition (2.3), we find an equation for the motion of the phase-transition boundary:

$$Q_* = \frac{2\mu}{\pi} H_0^2 \frac{E_2(\alpha^2/2\eta^2)}{[E_1(\alpha^2/4\eta^2)]^2 \alpha^2/2\eta^2}. \tag{2.9}$$

Using asymptotic expressions for the functions $E_1(z)$ and $E_2(z)$ [6], we find the limiting magnetic fields:

$$H_{max} = (2\pi Q_*/\mu)^{1/2} \quad (\alpha \to \infty). \tag{2.10}$$

Equations (2.4) and (2.8) are used to solve the problem of the diffusion of a superintense magnetic field from an aperture of finite radius R. For this purpose, we must specify at $r = R$ and at initial time t_0 the magnetic-field and heat-evolution distributions given by Eqs. (2.4) and (2.8), with an account of (2.3) and (2.9) and under the condition

H (R, t_0) = H_0. For the limiting values H_{max}, the introduction of the new initial condition is evidently inconsequential, so (2.10) remains valid.

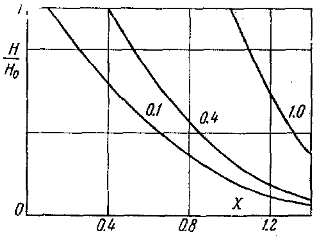

Fig. 1

3. **Comparison with experimental data.** Figures 1 and 2 show the magnetic-field and heat-evolution distributions with respect to depth for various velocities $X = x/2\eta t^{1/2}$ of the phase-transition boundary during the diffusion of the magnetic field into the half-space. The curves correspond to the values $\alpha/2\eta = 0.1, 0.4, 1.0$, as indicated in the figures. The slopes of the distributions are seen to increase with increasing α. Presumably, therefore, at a sufficiently large α the magnetic field diffuses into the metal at the velocity of the phase-transition boundary, and hardly passes the latter.

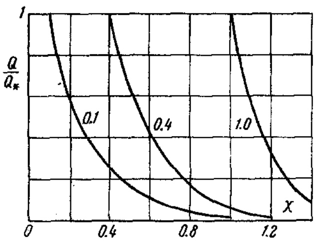

Fig. 2

Figure 3 shows the dependence of the field propagation velocity on the quantity $H^* = H_0/(\pi Q_*/\mu)^{1/2}$, given by (1.14), is approached, the magnetic field begins to propagate into the metal much more rapidly. This means that as the magnetic field source increases in intensity, field leakage increases even more rapidly, and H_{max} is not reached.

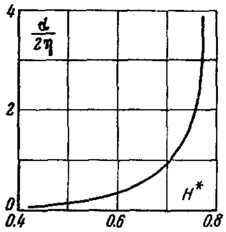

Fig. 3

Let us evaluate the limiting magnetic field which can be reached in an apparatus with plane metallic boundaries. We evidently have

$$Q_* \sim s(T_* - T_0) + \lambda + q, \tag{3.1}$$

where s is the heat capacity, q is the heat of fusion, λ is the latent heat of vaporization per unit volume of the metal, and T_0 is the initial temperature. For copper, we have s ~ 6 cal/mole·deg and λ ~ 73 kcal/mole; at $T_* \sim 2600°$ C we find from (3.1) and (1.14) that $H_{max} \sim 2.9 \cdot 10^8$ a/m (3.6 MOe). We note that the values of Q_* and H_{max} may be sensitive to the vapor pressure at the phase-transition boundary.

The magnetic pressure P_n in the region at which the phase-transition has occurred is zero, since it is a consequence of ponderomotive forces:

$$P_n(x, t) = \mu \int_0^x H(x, t) \frac{\partial H}{\partial x}\, dx.$$

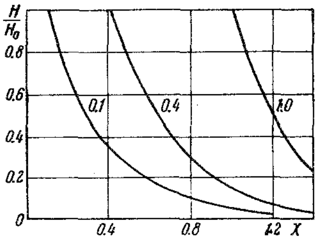

Fig. 4

In experiments in cavity and coaxial magnetic-compression generators [7–10], magnetic fields up to 1.6 MOe have been achieved, regardless of the linear dimensions, the initial magnetic fields and currents, or the compression rates. In one case, a field of 2.6 MOe was reported [11].

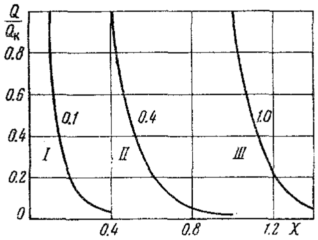

Fig. 5

Figures 4 and 5 show the magnetic-field and heat-evolution distributions with respect to depth for various transition-boundary velocities during the diffusion of a concentrated axial magnetic field held constant at a superintense level. The curves are seen to have a greater slope than in the case of field diffusion into a half-space.

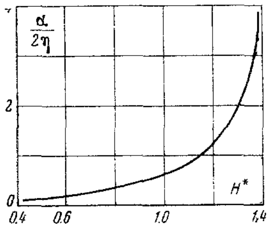

Fig. 6

Figure 6 shows the dependence of the transition-boundary velocity on the magnetic field. Using (3.1) and (2.10) for the limiting magnetic field intensity, we have the following for copper shells:

$$H_{\max} \sim 5.1 \cdot 10^8 \text{ A/m } (6.4 \text{ MOe}).$$

Magnetic fields up to 1.6 MOe have been produced experimentally through the discharge of a capacitor battery into a single-turn solenoid of copper, bronze, or hardened steel [12]. In this case, however, the solenoid was fitted with a planar radial gap through which the magnetic field entered. According to (2.10) the magnetic field in the gap, and thus that in the solenoid, cannot exceed 3 MOe; this is close to the experimentally observed value.

The agreement between the limiting field intensities in solenoids of various materials having initial conductivities differing by a factor of at least ten can be explained by the fact that, according to (1.14) and (2.10), the limiting magnetic field depends not on the conductivity but on the heat of vaporization Q_*, which is roughly the same for the materials used. In axially symmetric magnetic-compression generators, it is a simple matter to produce magnetic fields up to 4–5 MOe [9,13–15], close to the value given by (2.10).

Only in exceptional experiments can magnetic fields of 14–25 MOe be produced [7,8,16].

In contrast with these systems, in an axially symmetric magnetic-compression generator high pressures may arise in the region in which the phase transition has occurred and at the phase-transition boundary, because of shell compression. Let us evaluate the pressure P which arises in an ideally incompressible shell of density ρ. We are primarily interested in the pressure in the region which has undergone the phase transition and at its boundaries, where there are no volume ponderomotive forces. In this region the hydrodynamic equations for an incompressible liquid are

$$\frac{\partial rv}{\partial r}=0, \qquad \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial r} \tag{3.2}$$

where r is a cylindrical coordinate.

It has been established experimentally [9, 16] that the inner boundary of the shell moves at a constant velocity v_b for nearly the entire time interval until it stops completely; then we have $r_b = r_0 - v_b t$, where r_0 and r_b are the initial and instantaneous radii, respectively, of the inner boundary of the shell. Since the inner boundary of the shell is a free surface, we have $P(r_b, t) = 0$.

Under these conditions the solution of system (3.2) is

$$P = \rho v_b^2[\ln r / r_b + 1/2\,(1 - r_b^2/r^2)]$$

For copper, we have $\rho = 8.9 \cdot 10^3$ kg/m^3; at $v_b = 3000$ m/sec and $r_b/r \sim 0.5$, we find from this equation that P = 800 kbar, considerably above the critical pressures.

It follows that in an axially symmetric magnetic-compression generator, the phase-transition boundary stops after it reaches a depth at which the pressure is equal to the critical pressure; if the pressure continues to increase, this boundary starts to move in the opposite direction. Presumably, therefore, as the magnetic field intensity increases, the depth to which it diffuses increases; however, there is a simultaneous increase in the pressure near the phase-transition boundary, so the magnetic field may ultimately exceed the estimate in (2.10).

Experimental studies confirm this assumption. For roughly identical geometric and energetic conditions, H_{max} values of 2–3 MOe [16], 3.7 MOe [15], and 14.3 MOe [16] have been obtained at initial magnetic fields of 25, 33, and 90 kOe, respectively.

A field of $H_{max} \sim 25$ MOe has been reported at $v_b \sim 10 - 20$ km/sec [7, 8]. In this case the initial magnetic field, produced by a solenoidal magnetic-compression generator, was apparently greater than 90 kOe.

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